Topic Study Group 56

Philosophy of Mathematics Education

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1. Introduction

The philosophy of mathematics education can be traced back to the work of Plato. In his Republic Plato considered deeply the role and purpose of mathematics in teaching and learning. His enquiries were founded on ethics, for questions of meaning and purpose within a social context inevitably bring in the Good. At the same time, he was interested in the epistemology and ontology of mathematics and its relations with the Truth and Beauty. Overall, Plato displayed great interest in the subject of mathematics throughout his philosophical work, and he is an inspirational godfather and patron saint of the philosophy of mathematics education.

Current mathematics education research is mostly concerned with two questions, one epistemological and the second methodological. The epistemological questions ask what is mathematical truth and how do we justify and explain it, and above all, how to we come to know it? The methodological questions concern how we can best and most effectively teach and facilitate the learning of mathematics. Research in the philosophy of mathematics education also addresses epistemological questions of mathematics and its teaching and learning, but it does so more explicitly, more theoretically. In addition, it considers the ontological, aesthetic and ethical issues of mathematics with respect to education and society.

The philosophy of mathematics education is an interdisciplinary area of research that incorporates many questions.

- What are the goals and purposes of mathematics education?
- What can we learn from deep analyses of the methods and means of teaching
- and learning mathematics, as well as from studying the underlying theories and philosophies?
- What new insights are revealed by the application of deep theoretical approaches including Phenomenology, Hermeneutics, Complexity, Embodiment and Critical Theory within research in the philosophy of mathematics education?

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- What are the relationships between and, the mutual influences on each other, of the philosophy of mathematics and mathematics education?
- How do the personal philosophies of mathematics and mathematics education of learners, teachers, teacher educators and researchers impact on practice?
- How are the different actors of interest including students, teachers, researchers, theorists, philosophers and mathematicians linked together professionally within the fields of mathematics education research and practice?
- How do mathematics and the philosophy of mathematics impact on the nature, structure and content of mathematics for teaching?
- What do deep analyses of mathematics itself tell us about its structures, processes and fundamental concepts and about their relationships with its teaching and learning?

2. General Information

Due to the pandemic, there were only 14 presentations out of 24 originally submitted. However, all papers accepted to the original conference have been published in the Special Issue of the Mathematics Teaching Journal Online Vol. 12 N 2 at https://commons.hostos.cuny.edu/mtrj/archives/volume-12-n-2/

Also, many of the authors have been invited to contribute to the upcoming volume Philosophy of Mathematics Education: Work in Progress to be published by Springer in 2023.

The presentations were divided in accordance with the 2020 plans, however we had to fill up a couple of important spots in the program. Scovsmose was especially invited to give an introductory talk on Mathematics and Ethics.

Our surviving presentations have arranged themselves loosely along three themes:

1. Philosophical foundations and approaches, and within them we have several presentations touching upon critical mathematics education; we have an example of phenomenological approach (we had several more examples in the original collection) as well as attempts at epistemological clarification.
2. Philosophical problems; formulating and solving philosophical problems. For example, two presentations following Mathematics and Ethics, address the problem of rigor and the problem of imagination. On Friday we have addressed the problem of the algorithm and the problem of appropriation.
3. Philosophy of mathematics classroom. Here we also have a variety of lenses, math classroom epistemologies, digital games or theory-practice divide. Unfortunately, several other articles addressing modern Internet classrooms have also disappeared.

Meeting of TSG-56 took place on
Tuesday July 13, 19:30 – 21:00
Friday July 16, 21:30 – 23:00
Saturday July 17, 14:30 – 16:30

3. List of Presentations

Tab. 1. The list of papers and posters presented

Paper and author(s)
Session 1
[1] Mathematics and ethics. ***Ole Scovsmose*** (Brazil)
[2] Philosophy, rigor and axiomatics in mathematics: intimately related or imposed? ***Min Bahadur Shrestha*** (Nepal).
[3] Imagination in the philosophy of mathematics and its implication for mathematics education. ***Yenealem Ayalev*** (Ethiopia).
Session 2
[4] Towards a philosophy of algorithms as an element of mathematics education. *Regina D. Möller and* ***Peter Collignon*** (Germany).
[5] Appropriation mediates between social and individual aspects of mathematics education. ***Mitsuru Matsushima*** (Japan).
[6] Philosophical inquiry for critical mathematics education. ***Nadia Kennedy*** (USA).
[7] Towards critical mathematics. ***Theodore Savich*** (USA).
[8] Recognizing mathematical anthropocentrism. ***Thomas Ricks*** (USA).
[9] Curriculum system of the philosophy of mathematics education for normal students. ***Yaqiang Yan****, Suyue Xue, and Junfeng Ma* (China).
Session 3
[10] Research procedures to understand algebraic structures: hermeneutic approach. ***Maria Bicudo*** (Brazil).
[11] $\mathbf{2+2=4}$? mathematics lost between two pitfalls of essentialism and alternative truths. ***David Kolosche*** (Austria).
[12] Does constructivism tell us how to teach? ***Bronislaw Czarnocha*** (USA).
[13] Teachers epistemology on the origin of mathematical knowledge. ***Karla Sepúlveda Obreque*** *and Javier Lezama Andalón* (Chile).
[14] Mathematical education, body and digital games: play the ball in this way so that it goes, it goes further than the floor. ***Maurício Rosa****, Danyal Farsani, and Caroline Antunes da Silva* (Brazil).
[15] Internet, teaching mathematics: weaving the web. ***Marli Regina dos Santos*** (Brazil).

The first two papers in session 1 studied the underlying theories and philosophies. Scovsmose' study[1] based on the theory of four-dimensional philosophy of mathematics: ontological, epistemological, sociological and ethical. Ethical dimension is explored concentratedly here by showing the broad range of social implications set in motion through bringing mathematics into action. These implications are illustrated in terms of quantifying, digitalizing, serializing, categorizing and imagining. Conclusion draws that the philosophy of mathematics can bring mathematical expertise out of the ethical vacuum. Shrestha[2] examined how philosophy, rigor and axiomatic are related. It seems that philosophy has distant but determining impression on the nature of mathematical knowledge, but rigor and axiomatics seems to be more internal to mathematics.

Most studies in this group focused on specific mathematics education, with different research directions. Ayalev[3] considered "imagination" as a subtopic under mathematics education. Besides the argument whether mathematics was invented or discovered, the construct "imagination" was discussed for the learning of Mathematics.

Möller and Collignon[4] discussed algorithms as well as their roles and importance for mathematics educaiton from a philosophical point of view. The framework of (post-)modernism and a constructivist approach were used. As for social and individual aspects of mathematics education, Matsushima[5] identified five appropriation stages from the discussion of a structural model of social constructivism based on a socio-cultural approach in mathematics education. The result of analysis revealed that a gap in appropriation could occur during the process, and that gap could become the source of creativity. Ricks[8] suggested the benefits of post-anthropocentrism for mathematics education considering that anthropocentric perspectives were inaccurate in lieu of many scientific findings about the mathematical abilities of many non-human entities. The ninth paper is the only one start from teacher education. Yan et al.[9] suggested a curriculum system for the philosophy of mathematics education for Chinese normal students. It is expected to provide "readable materials" for direct application in the practice of future teachers. Czarnocha's study[12] focused on the interface between theory and teaching practice of constructivism. The presentation argued that the research tool, constructivist teaching experiment did define the constructivist teaching methodology and through mathematics teaching-research, it could be introduced into mathematics classroom at large.

There are two presentations about critical studies. Kennedy[6] reported that philosophical inquiry could both offer a space for critical reflection on mathematics, for the development of an epistemological approach, and also a space for the deconstruction and reconstruction of beliefs about mathematics as a form of knowledge, about the social value of mathematical practice, and beliefs about oneself as a mathematics learner/thinker. In Savich's study[7], necessary conditions for arithmetic were expressed as material inferential rules using a normative vocabulary of commitments and entitlements. The explicated critical arithmetic is also related to other projects in critical mathematics education.

The tenth, eleventh and thirteenth paper are about mathematics and the philosophy of mathematics. Bicudo's study[10] was about algebraic and hermeneutic procedures. The openness of abstract algebra may happen through hermeneutic procedures. Kolosche's research[11] started from two examples from popular media to problematize the essentialist epistemologies and relativist epistemologies of mathematical knowledge. Obreque and Andalón[13] argued teachers epistemology attributed to the origin of mathematical knowledge under socioepistemological theory of educational mathematics.

The last two papers of this group are of digital technologies. Rosa et al.[14] conducted a study that high school students conjectured a way mathematically to improve their performance in an the electronic bowling game on Xbox One with Kinect. From that they are drawing up on embodied cognition articulated with the conceptions of perception and body-proper arising from the phenomenological view discussed by Merleau-Ponty. The conclusion is students' perception is shown by the acts of being-

with, thinking-with and knowing-doing-mathematically-with-Digital-Technologies. Santos[15] proposed a theoretical and philosophical reflection on the nature of the cyberspace to find the possibilities which the Internet opens to the teaching and learning of Mathematics. The process which educators created resources and spaces for pedagogical practice of mathematics on the Internet is explored.